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$$\begin{array}{ll} \alpha_1 = 360^\circ - 15^\circ\, 45'\, 25'', & \theta_0 = 90^\circ + 52^\circ\, 45'\, 48'', \\ \alpha_2 = 180^\circ - 75^\circ\ \ 6'\, 19'', & \theta_1 = 90^\circ + 44^\circ\ \ 5'\, 13'', \\ & \theta_2 = 90^\circ + 37^\circ\ \ 1'\, 51''. \end{array}$$

Whence,

$$\begin{array}{lllll} & \sin\alpha_1 = -.271557, & \cos\alpha_1 = .962422, & \sin\theta_0 = .605109, & \cos\theta_0 = -.796143, \\ (4) & \sin\alpha_2 = .966400, & \cos\alpha_2 = -.257044, & \sin\theta_1 = .718285, & \cos\theta_1 = -.695749, \\ & & & \sin\theta_2 = .798312, & \cos\theta_2 = -.602245. \end{array}$$

If we substitute these numerical values in the first equation of (3) the left side becomes

$$.271557(-\tfrac{1}{3} \times .796143 + \tfrac{1}{4} \times .695749) + .962422(\tfrac{1}{3} \times .605109 - \tfrac{1}{4} \times .718285) = .003532.$$

Substituting the values (4) in the second equation of (3), we get for its left side

$$-.966400(-\tfrac{1}{4} \times .695750 + \tfrac{1}{3} \times .602245) - .257044(\tfrac{1}{4} \times .718285 - \tfrac{1}{3} \times .798312) = -.003668.$$

Thus it is seen that the values (4) approximately satisfy conditions (1).

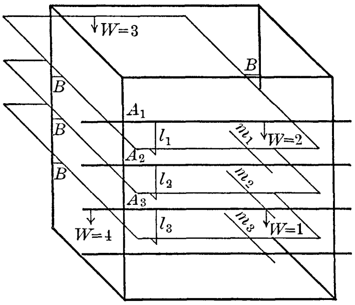
It is to be noted that in Professor Uhler's solution use is made of the law of refraction. Since this law is a special case of the Bliss-Mason condition, it follows that if Professor Uhler's results are used, the foregoing serves as a check on his solution.

II. Relating to an Equation Balance.

By E. L. Rees, University of Kentucky.

The equation balance described below, though less simple in design and construction than some others, offers, in the opinion of the writer, many compensating advantages.

This machine consists of a frame supporting two sets of levers. The levers of one set, which we shall call the scale levers, have their fulcrums at the points marked A in the figure. Just back of, and on a level with, each scale lever, except the top one, is a platform lever supported at its center on the knife edges marked B. These two sets of levers are connected in the following manner: A point one unit from the fulcrum on the first scale lever is joined to the first platform lever by a connecting rod l_1; similarly the second scale lever is connected to the second platform lever, etc. On each platform lever is a movable arm (m_1, m_2, m_3) resting on the corresponding scale lever. This arm passes through a slot in the scale lever so that all vertical forces may be transmitted through the arm to the scale lever.



The following example will serve to illustrate both the principle and the operation of the machine.

Let us solve the cubic equation $2x^3 - 3x^2 + x - 4 = 0$. Having balanced all the levers, we place on the first scale lever a weight 2, representing the first coefficient, in the equation. We then place the movable arms (m_1, m_2, m_3) in the same vertical line with this weight. Calling its lever arm x, we see from the moments that a force of $2x$ is transmitted through the connecting rod l_1, and the arm m_1, to the point immediately below on the second scale lever. We now place a weight 3, representing the second coefficient, on the rear end of the first platform lever so that it will exert a negative or upward force on the scale lever. We now have a force $2x - 3$ applied to the second scale lever at a distance x from the fulcrum. This in turn is transferred to the third scale lever as a force of $(2x - 3)x$ to which we add a force of one unit (third coefficient) by attaching a weight 1 to the arm m_2 at this point. We also place a weight 4 (constant in equation) on the third scale lever one unit to the left of the fulcrum since the constant is negative. The resulting moment will then be $[(2x - 3)x + 1]x - 4 = 2x^3 - 3x^2 + x - 4$. We now move the weights and arms back and forth on the scale levers, keeping them in the same vertical line, until the levers are balanced. The moment will then be zero and the length of the lever arm, which is seen to be the root of the equation, may be read from the scale on the first lever. This of course gives us only positive roots. Negative roots may be found by changing the signs of the roots of the equation.

Obviously the number of scale levers corresponds to the degree of the equation of highest degree that can be solved. The figure represents a balance which may be used for solving quadratics, cubics, and quartics. In this machine the unit was taken as two inches and the working interval from zero to ten inches, thus giving roots up to five units. As the mechanical details have not yet been perfected only the general plan of construction has been given.

The satisfactory results obtained from this crudely constructed balance with which the writer experimented give assurance that a carefully constructed machine, with proper lengths of levers and unit determined experimentally, will yield results of considerable precision.

III. Relating to the Real Locus Defined by the Equation $x^y = y^x$.

By Philip Franklin, College of the City of New York.

In the September number of the Monthly (Vol. XXIII, pp. 233–237) in a discussion of the real locus defined by the equation $x^y = y^x$, E. J. Moulton raised the question of the continuity of the locus in the third quadrant.

Having met this function in another connection, I was led to consider it in the following manner:

Putting $y = mx$ in $x^y = y^x$, $x^m = mx$, from which

$$x = m^{1/(m-1)} \quad \text{and} \quad y = m^{m/(m-1)}.$$

The curve is easily plotted from these equations or from the equation derived from them, $r = \sec \theta \tan \theta^{1/\tan(\theta - 1)}$.

In the first quadrant the curve is continuous since we obtain points on the curve for all positive values of m.

In the second and fourth quadrants the curve is discontinuous, since, in general, the $(m - 1)$th root of a negative number does not have a real value.

In the third quadrant points are obtained only for those values of m for which $m^{1/(m-1)}$ has a real negative value, which is not true in all cases; *e. g.*, when m is an even integer. The curve is, therefore, discontinuous in the third quadrant.